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Understanding the Pandemic: Gaussian Curve and Bernoulli's Equation

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ABSTRACT:

Understanding the nature through the language of science has made several progressive impact to the humankind. From micro organisms to macro organisms many research ideas are well recorded in literature. The mathematical expression of many biological phenomenon have played a vital role to the society. One such efficient tool to express biological process is Differential Equations. In this paper, we study two models namely: Gaussian curve to describe the epidemic and the Bernoulli's model to describe population growth affected by pandemic. Also we make the qualitative analysis of the infections caused by COVID-19 and we give an alternative model to the Bernoulli's model.

Key words: Bernoulli's model, Gaussian Curve, COVID-19.

1. INTRODUCTION:

Nature nurtures us and it resides many hidden secrets. We the human beings, the rational animal are just the seekers of the nature. Wellness of the nature is our primary responsibility. In the history, there are many instances where the nature alarms and sanctifies itself. The fact is, the wellness of an individual depends on the wellness within and the wellness of the society. Health is one of the major factor that defines wellness. In 21^{st} century, the world has witnessed the pandemic situation caused by novel Corona-virus. This epidemic affected the globe both socially and economically. It stood substantial making this *Blue Ball* vulnerable almost everywhere. All round development of any nation depends on the development of the society in every aspects. The societal demand in the context of development is challenging. To have a prosperous society, the stakeholders need to focus on the benefits of the citizens. In reaching a layman, one must know *population density* and the expectations of the society. Henceforth, its important to know the population growth. As a helping tool, we have one such model called *population model* that gives the idea of progression of the human kind.

Mathematical Modelling refers to the formulation of empirical or observed facts using suitable definitions in a eloquent way. There are many models that are available in the literature namely : nuclear fission model, linear models in economics, predator-prey model, epidemic models, stock market forecast models and so on [4, 7, 9–11]. They help in understanding and analysing the system in context. The conclusion drawn out from them after a detailed study will help in formulating the allied concepts with vigilance. In this paper, we give mathematical formulation of epidemic and population growth. As these two factors are progressive they exhibits the constant change. Its worth noting that the rate of change can be expressed by one of the language of mathematics termed as *differential equations*. The work in this paper is two fold, we give two models: Gaussian curve (Gc) which models the spread of epidemic and the

Bernoulli's model (Bm) to express the population growth after infections. In section 2 we express the Gc model and qualitative analysis of Gc. In the concluding, we discuss the population growth given by Bm and give an autonomous system that agrees with Bm.

2. UNDERSTANDING THE RATE OF INFECTIONS

2.1. Gaussian Curve: Revealing the Growth of Pandemic COVID-19

In this section, we discuss the number of infections of COVID-19 in India between December 2019 to February 2023. We give the model namely, Gaussian curve which is suitable for the rate of change of infections. Finally we make qualitative analysis of the infections that varied accordingly with time.

Let $y(t)$ be the function of time t and $\frac{dy}{dt}$ be the rate of change of infection at given small interval of time δt.

Let $\beta(t_i)$ be the number of infections at given time t_i. The maximum of $\beta(t_i)$ be $\alpha(t)$ and c be the duration of impact of $\alpha(t)$. As the pandemic cases is directly proportional to an exponential function, it may increase or decay. Consider the linear ordinary differential equation of first order

$$y + \frac{c^2}{t-\beta(t)} y^l = 0; \quad t \neq \beta(t) \tag{1}$$

Solution of (1),

$$y(t) = \alpha(t)\, e^{\frac{(t-\beta(t))^2}{2c^2}} \tag{2}$$

For more details please see [9].

2.1.1. Qualitative Study

In this subsection, we make the qualitative analysis of the infected covid cases in India. And we finally compare the solution (2) with the graph of actual data available.

Month and Year	**Cases**
02-02-2020	0
09-03-2020	68
23-03-2020	619
13-04-2020	7356
27-04-2020	13484
04-05-2020	22959
18-05-2020	40951
25-05-2020	50275
08-06-2020	74294
15-06-2020	89539
22-06-2020	118398
06-07-2020	176388
20-07-2020	307904
03-08-2020	402287
17-08-2020	455258
31-08-2020	571078

07-09-2020	640545
21-09-2020	591913
28-09-2020	556841
05-10-2020	504433
19-10-2020	370260
26-10-2020	319271
09-11-2020	306825
23-11-2020	297133
07-12-2020	212807
21-12-2020	174194
28-12-2020	136115
12-01-2021	12584
25-01-2021	91650
08-02-2021	78577
15-02-2021	86711
22-02-2021	105080
08-03-2021	148249
22-03-2021	372494
05-04-2021	873296
12-04-2021	1429304
19-04-2021	2172603
26-04-2021	2597285
03-05-2021	2738957
10-05-2021	2387663
31-05-2021	914539
07-06-2021	630650
21-06-2021	351218
05-07-2021	291789
26-07-2021	283923
02-08-2021	278637
16-08-2021	231658
23-08-2021	283923
06-09-2021	248248
20-09-2021	204580
27-09-2021	161158

11-10-2021	114244
25-10-2021	97832
08-11-2021	81771
15-11-2021	73106
06-12-2021	57255
13-12-2021	49765
27-12-2021	102330
03-01-2022	638872
17-01-2022	2115100
14-02-2022	191052
28-02-2022	46836
07-03-2022	28038
14-03-2022	16852
21-03-2022	11612
28-03-2022	8672
11-04-2022	6826
25-04-2022	21643
02-05-2022	23006
23-05-2022	1667
13-06-2022	74675
27-06-2022	112465
11-07-2022	127948
27-07-2022	131056
01-08-2022	125921
15-08-2022	85965
22-08-2022	68073
05-09-2022	38824
26-09-2022	26373
10-10-2022	16815
24-10-2022	9524
21-11-2022	2547
26-12-2022	1543

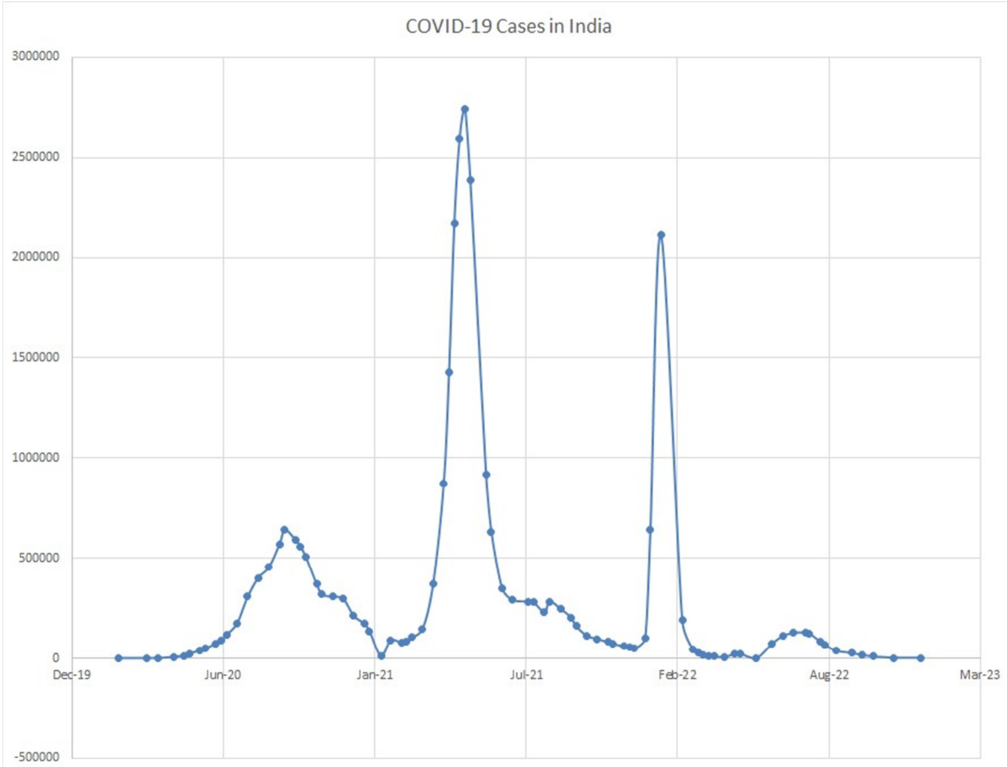


Figure 1.

Figure 1 is the graph for the observed cases of the above Table.

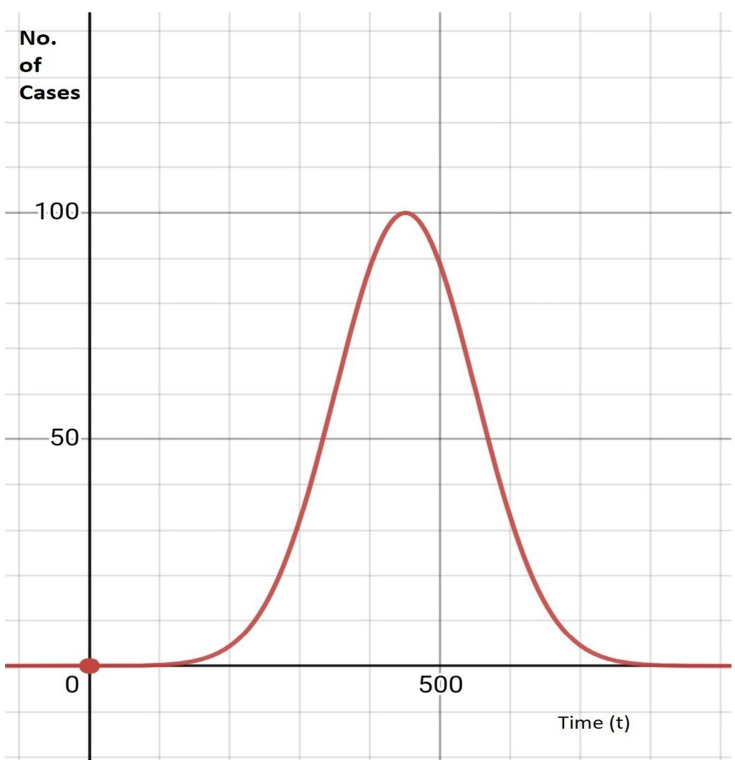


Figure 2.

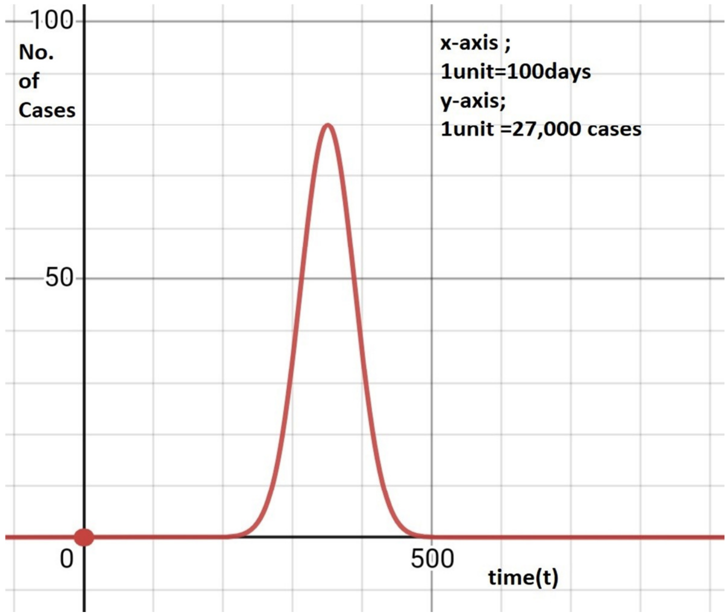


Figure 3.

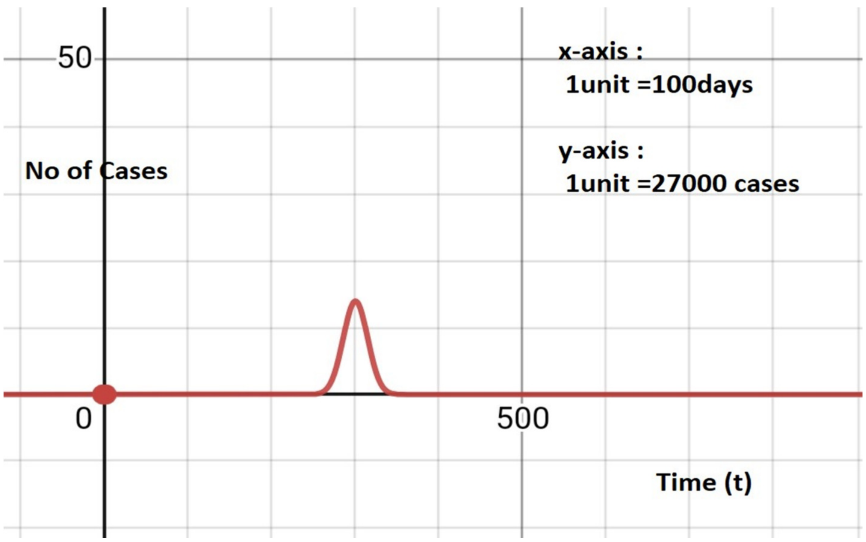


Figure 4.

Figure 2 -Figure 4 are the solutions (1) at different time.

Thus, we make the following observations

- The solution (2) has the real c which denotes the width of the function.
- The amplitude of the function is $\alpha(t)$ which matches with the maximum cases given in the Table.

For more details please refer [3, 9].

2.2. Population Growth

2.3. Bernoulli's Model: Expressing Population Growth

In this section, we describe the population growth using Bernoulli's equation. We give an autonomous differential equation that agrees with Bernoulli's model and analyse its solution.

2.3.1. Expression for Bernoulli's Model

The Bernoulli's Model (Bm) was formulated by Daniel Bernoulli to express the effect of small pox infections to the population [15]. Now, we demonstrate the Bm to the pandemic outbreak COVID-19. Before that we make few preliminary assumptions as given below.

Let η be the probability of people lose their life after infection and thus their survival probability will be $1-\eta$. Let ξ be the probability of an individual being infected in a day. Hence the probability of an individual getting infected between x and $x+\delta x$ is $\xi \cdot \delta x$

Let $C(x)$ be the number of people suspected at a day x without the infection, $D(x)$ be the number of people who are not affected by infection at day x. Hence number of people alive on a day x is given by $P(x) = C(x) + D(x)$.

Let $M(x)$ be the mortality at the age x for some reason other than the COVID-19, then $M(x)dx$ denote the probability of mortality between x and $x + \delta x$. Now, we formulate Bm [15] to obtain,

$$\frac{dC(x)}{dx} = -\xi C(x) - M(x)C(x) \tag{3}$$

$$\frac{dD(x)}{dx} = \xi(1-\eta)C(x) - M(x)D(x) \tag{4}$$

Adding (3) and (4) we obtain

$$\frac{d(C(x)+D(x))}{dx} = -\eta\xi(x) - M(x)(C(x)+D(x))$$

$$\frac{dP(x)}{dx} = -\eta\xi(x) - M(x)(P(x)) \tag{5}$$

By eliminating $M(x)$ from (3) and (5) we get

.

$$-M(x) = \xi + \frac{1}{C(x)}\frac{dC(x)}{dx} = \eta\xi\frac{\xi(x)}{P(x)} + \frac{1}{P(x)}\frac{dP(x)}{dx}$$

$$\frac{1}{P(x)}\frac{dC(x)}{dx} - \frac{C(x)}{P(x)^2}\frac{dP(x)}{dx} = -\xi\frac{C(x)}{P(x)} - \eta\xi\left(\frac{C(x)}{P(x)}\right)^2$$

$$\frac{d}{dx}\frac{C(x)}{P(x)} = -\xi\frac{C(x)}{P(x)} + \eta\xi\left(\frac{C(x)}{P(x)}\right)^2$$

By substituting $\frac{C(x)}{P(x)} = F(x)$ we obtain,

$$F^I = -\xi F + \eta\xi F^2 \tag{6}$$

which is well known Bernoulli's Equation.

Equation (6) is equivalent to $G^I = \xi G - \eta\xi$, where $\frac{1}{F} = G$.

By setting $G(0) = \frac{1}{F(0)}$ and $H(x) = G(x) - \eta$

We compute that $H(x) = (1-\eta)e^{\xi x}$.

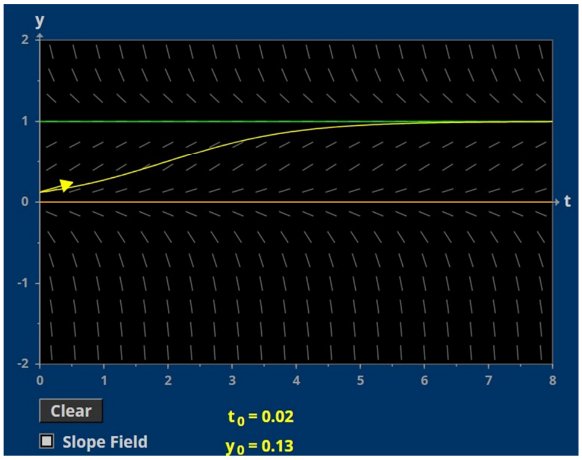
Thus, $G(x) = (1-\eta)e^{\xi x} + \eta$ implies that

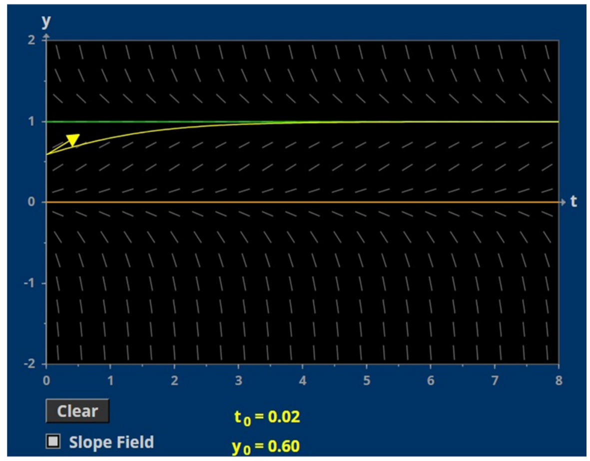
$$F(x) = \frac{1}{(1-\eta)e^{\xi x} + \eta} \tag{7}$$

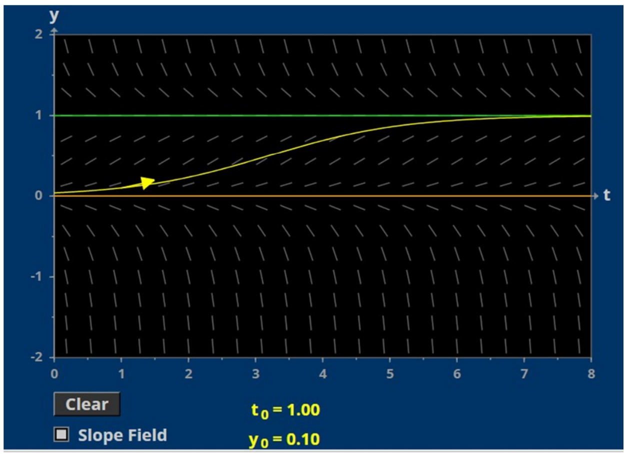
is the solution to (6).

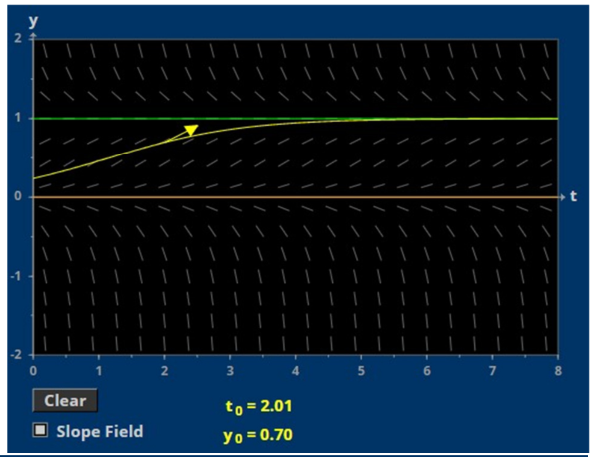
2.3.2. Plots of the solution

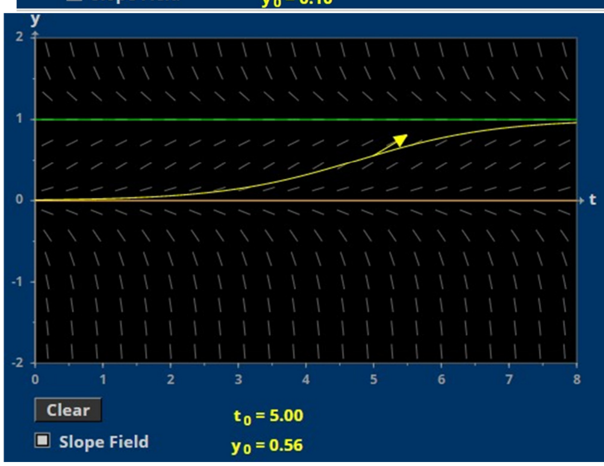
Using mathematical packages we plot the graph of $F(t)$ for different values of ξ ,η and t.

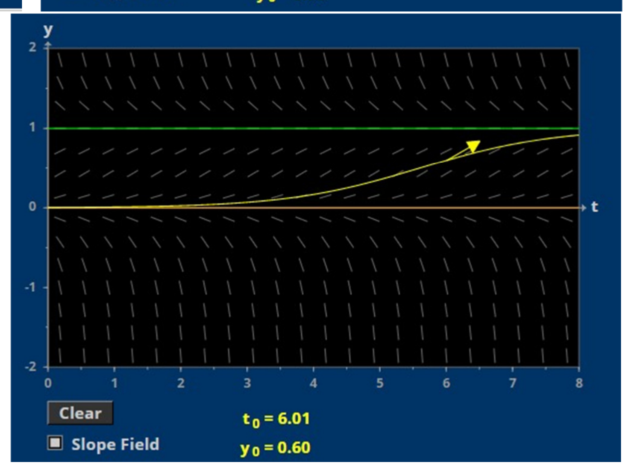












2.4. Autonomous Equation describing the Population growth

In this subsection we give alternative model through an autonomous differential equation [1, 4, 7].

Consider the ODE

$$\frac{dy}{dt} = -(y-k)(y-m); \tag{8}$$

with the initial condition $y(a) = l$ and $0 < k < l < m$.

Critical points of (8) are $y = k$ and $y = m$.

Hence, the solution y(t) lies in the strip of k and m.

In the positive neighbourhood of $l : y = l + \epsilon$ where $\epsilon > 0$ we have

.
$$\frac{dy}{dt} = -(l+\epsilon-k)(l+\epsilon-m) > 0$$

⇒ $y(t)$ is increasing.

In the negative neighbourhood of $l : y = l - \epsilon$ where $\epsilon > 0$ we have

$$\frac{dy}{dt} = -(l-\epsilon-k)(l-\epsilon-m) > 0$$

⇒ $y(t)$ is increasing. By the existence and uniqueness of the solution to first order differential equation [7], equation (8) is nothing but the bounded increasing solution which agrees with the (6).

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