

A CONTRIBUTION TO THE STUDY ON CHAOS SPACE IN FUZZY ROUGH ALGEBRAIC TM SYSTEM

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ABSTRACT

This paper introduces a new concept called chaos space in fuzzy rough algebraic TM system. It studies the properties of chaos space and introduces a new system called fuzzy rough chaotic structure space. It emphasizes the properties of chaotic interior and chaotic closure.

Keywords: fuzzy rough chaotic TM algebra, fuzzy rough algebraic chaos system, fuzzy rough algebraic chaotic closed, fuzzy rough algebraic chaotic open

1. Introduction

By R. L. Devaney [1], the idea of chaos in generic metric space was first proposed. Devaney's concept of chaos' foundational characteristics was formed. Additionally, the characteristics of chaos were developed and researched [3,4,5]. This chapter deals with the chaos space in fuzzy rough algebraic TM system. Some of its interesting properties were discussed.

2. Preliminaries:

Definition 2.1 [3]

Let $f: X \to X$ be any mapping and let X be a nonempty set. Give any fuzzy set in X, λ . Under the mapping f, the fuzzy orbit $O_f(\lambda)$ of λ is defined as $O_f(\lambda) = \{\lambda, f(\lambda), f^2(\lambda), ...\}$

Definition 2.2 [2]

Let $f: X \to X$ be any mapping and let X be a non-empty set. The intersection of all members of $O_f(\lambda)$ is defined as $FO_f(\lambda) = \{\lambda \land f(\lambda) \land f^2(\lambda) \land ...\}$, which represents the fuzzy orbit set of λ under the mapping f.

Definition 2.3 [3]

Let (X, τ) be a fuzzy topological space. Let any mapping $f: X \to X$. Fuzzy orbit open set under the mapping f is the fuzzy orbit set under the mapping f that is in fuzzy topology τ . Under the mapping f, its complement is referred to as a fuzzy orbit closed set.

Definition 2.4 [5]

Let $f: X \to X$ be any mapping and let X be a nonempty set. If $f^n(\lambda) = \lambda$, for some $n \in Z_+$, then a fuzzy set λ of X is called a fuzzy periodic set with regard to f. Fuzzy periodic of X is the smallest of these n. **Definition 2.5 [4]**

Let $f: X \to X$ be any mapping and let (X, τ) be a fuzzy topological space. Fuzzy-periodic open set with respect to f is the fuzzy periodic set with respect to f in fuzzy topology τ . Fuzzy periodic closed set with respect to f is the name of its complement.

3. FUZZY ROUGH ALGEBRAIC CHAOTIC STRUCTURE SPACE

Definition 3.1

Let $A = (A_L, A_U)$ be fuzzy rough algebraic of a fuzzy rough algebraic TM system. Then the fuzzy rough algebraic TM orbit of an algebraic *B* is denoted and defined by $\mathcal{O}_{TM}(B)$ of *B* is a mapping $\mathfrak{f}: A \to A$ such that $\mathcal{O}_{TM}(B) = \{B, \mathfrak{f}(B), \mathfrak{f}^2(B), \dots, \}$.

Definition 3.2

For a fuzzy rough algebraic TM system, let $A = (A_L, A_U)$ be its fuzzy rough algebraic. Under the mapping f, the fuzzy rough orbit algebraic of *B* is given by $F_{RTM}\mathcal{O}_{TM}(B) = (B \cap f(B) \cap f^2(B) \cap ...)$, which is the intersection of all elements of $\mathcal{O}_{TM}(B)$.

Definition 3.3

Let *A* be a fuzzy rough algebraic, $f: A \to A$ be any mapping, and (X, TM) be a fuzzy rough algebraic TM system. Fuzzy rough algebraic open orbit is the fuzzy rough orbit algebraic under the mapping f that is in the fuzzy rough algebraic TM system. Under the mapping f, its complement is referred to as a fuzzy rough algebraic closed orbit.

Definition 3.4

A fuzzy rough algebraic *A* is called fuzzy rough periodic algebra of a fuzzy rough algebraic TM system with respect to the mapping $f: X \to X$ if $(f_{TM})^n(A) = A$, for least $n \in Z_+$.

Definition 3.5

Let $f: X \to X$ be any mapping and let (X, TM) be a fuzzy rough algebraic TM system. In the fuzzy rough algebraic TM system, the so-called fuzzy rough periodic algebra with respect to f is referred to as the fuzzy rough algebraic periodic open, and its complement is referred to as the fuzzy rough algebraic periodic closed.

Notation 3.1.1

 $F_{RTM}Pe = \bigcap \{ \text{fuzzy rough algebraic periodic open with respect to } \# \}.$

Definition 3.6

A fuzzy rough algebraic *A* of a fuzzy rough algebraic TM system (*X*, *TM*) is said to fuzzy rough algebraic compact if and only if for all $\mathcal{B} \subset TM$ such that $sup_{B \in \mathcal{B}} B \ge A$ and for all $\varepsilon > 0$, there exists a finite subalgebra $\mathcal{B}_0 \subset \mathcal{B}$ such that $sup_{B \in \mathcal{B}_0} B \ge A - \epsilon$. The collection of all algebraic compact is denoted by $F\mathcal{G}_{TM}(X)$.

Definition 3.7

A fuzzy rough algebraic A in a fuzzy rough algebraic TM system (X, TM) is called fuzzy rough algebraic dense if there exists no fuzzy rough closed algebraic B in (X, TM) such that A < B < 1. That is, $F_{RTM}cl(A) = \tilde{1}$ in (X, TM).

Definition 3.8

Let (X, TM) be a fuzzy rough algebraic TM system and $A \in F\mathcal{G}_{TM}(X)$. Then the mapping $\mathfrak{f}: X \to X$ is said to be fuzzy rough chaotic TM algebra with respect to A if

- (i) $F_{RTM}cl(F_{RTM}\mathcal{O}_{TM}(A)) = \tilde{1}$,
- (ii) $F_{RTM}Pe$ is fuzzy rough algebraic dense.

Notation 3.1.2

- (i) When *A* is a fuzzy rough algebraic in *X*, then $F_R \mathfrak{C}(A_{TM}) = \{ \mathfrak{f} : X \to X/\mathfrak{f} \text{ is a fuzzy rough chaotic TM algebra with respect to A}.$
- (ii) $F_R \mathfrak{CA}(X) = \{A \in F \mathcal{G}_{TM}(X) / F_R \mathfrak{C}(A_{TM}) \neq \tilde{0}.$

Definition 3.9

If $F_R \mathfrak{CA}(X) \neq \tilde{0}$, then a fuzzy rough algebraic TM system (X, \mathfrak{CA}_{TM}) is referred to as a fuzzy rough algebraic chaos system. The elements of $F_R \mathfrak{CA}(X)$ are referred to as fuzzy rough chaotic TM algebra in X if (X, TM) is a fuzzy rough algebraic chaos system.

Definition 3.10

Consider a fuzzy rough algebraic chaos system (X, \mathfrak{CA}_{TM}). Let \mathfrak{S} be the set of fuzzy rough chaotic TM algebras in X that meet the requirements listed below:

- (i) $\tilde{0}, \tilde{1} \in \mathfrak{S}$
- (ii) If $A_1, A_2 \in \mathfrak{S}$, then $A_1 \cap A_2 \in \mathfrak{S}$.
- (iii) If $\{A_j : j \in J\} \subset \mathfrak{S}$, then $\bigcup_{j \in J} A_j \in \mathfrak{S}$.

Then, in *X*, \mathfrak{S} is referred to as the fuzzy rough algebraic chaotic structure, and the fuzzy rough algebraic chaotic structure space is denoted as ($\mathfrak{S}, \mathfrak{CA}_{TM}$). Fuzzy rough algebraic chaotic closed is the complement of \mathfrak{S} , while fuzzy rough algebraic chaotic open is the set of components that make up \mathfrak{S} .

Definition 3.11

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Let A be chaotic TM algebra in X. Then

(i) The fuzzy rough chaotic interior of A is defined and denoted as

 $F_R \mathfrak{CA}_{TM} int(A)$

 $= \bigcup \{B: A \supseteq B, B \text{ is a fuzzy rough algebraic chaotic open} \}$

(ii) The fuzzy rough chaotic closure of A is defined and denoted as

 $F_R \mathfrak{CA}_{TM} cl(A)$

 $= \bigcap \{B: A \subseteq B, B \text{ is a fuzzy rough algebraic chaotic closed} \}$

Proposition 3.1.1

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Let *D* be chaotic TM algebra in *X*. Then the following statements hold:

- (i) $F_R \mathfrak{CA}_{TM} int(D)$ is a fuzzy rough algebraic chaotic open.
- (ii) $F_R \mathfrak{CA}_{TM} cl(D)$ is a fuzzy rough algebraic chaotic closed.
- (iii) *D* is a fuzzy rough algebraic chaotic open if and only if $D = F_R \mathfrak{GA}_{TM} int(D)$.

Proof

The proof follows from the Definition 3.11

Remark 3.1.1

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Let *D* be chaotic TM algebra in *X*. Then the following statements hold:

- (i) $F_R \mathfrak{CA}_{TM} int(D) \subseteq D \subseteq F_R \mathfrak{CA}_{TM} cl(D)$.
- (ii) $(F_R \mathfrak{CA}_{TM} int(D))' = F_R \mathfrak{CA}_{TM} cl(D)'$
- (iii) $(F_R \mathfrak{CA}_{TM} cl(D))' = F_R \mathfrak{CA}_{TM} int(D)'$

Proof

The proof is simple and obvious by *Proposition 3.1.1*.

Proposition 3.1.2

Let $\{D_j\}_{i \in J}$ be a family of chaotic TM algebra in X and J be an indexed set. Then for $j \in J$,

- (i) $\cup_j F_R \mathfrak{CA}_{TM} cl(A_j) \subseteq F_R \mathfrak{CA}_{TM} cl(\cup_j (A_j))$
- (ii) $\cup_{j} F_{R} \mathfrak{CA}_{TM} int(A_{j}) \subseteq F_{R} \mathfrak{CA}_{TM} int(\cup_{j} (A_{j}))$

Also, for any finite $n \in J$, $\bigcup_n F_R \mathfrak{CA}_{TM} cl(A_n) = F_R \mathfrak{CA}_{TM} cl(\bigcup_n (A_n))$.

Proof

The proof follows from *Definition 3.11*.

Proposition 3.1.3

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Let *D* and *E* be any two chaotic TM algebra in *X*. Then the following conditions hold:

- (i) $F_R \mathfrak{CA}_{TM} int(D) \subseteq D$.
- (ii) $D \subseteq E \Longrightarrow F_R \mathfrak{CA}_{TM} int(D) \subseteq F_R \mathfrak{CA}_{TM} int(E)$
- (iii) $F_R \mathfrak{CA}_{TM} int(F_R \mathfrak{CA}_{TM} int(D)) = F_R \mathfrak{CA}_{TM} int(D)$
- (iv) $F_R \mathfrak{CA}_{TM} int(D \cap E) = F_R \mathfrak{CA}_{TM} int(D) \cap F_R \mathfrak{CA}_{TM} int(E)$
- (v) $F_R \mathfrak{CA}_{TM} int(\tilde{0}) = \tilde{0}$
- (vi) $F_R \mathfrak{CA}_{TM} int(\tilde{1}) = \tilde{1}$

Proof

From Definition 3.10 and 3.11 the proposition is proved.

Proposition 3.1.4

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Let *A* and *B* be be any two chaotic TM algebra in *X*. Then the following conditions hold:

- (i) $A \subseteq F_R \mathfrak{CA}_{TM} cl(A)$.
- (ii) $A \subseteq B \Longrightarrow F_R \mathfrak{CA}_{TM} cl(A) \subseteq F_R \mathfrak{CA}_{TM} cl(B)$
- (iii) $F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} cl(A)) = F_R \mathfrak{CA}_{TM} cl(A)$
- (iv) $F_R \mathfrak{CA}_{TM} cl(A \cup B) = F_R \mathfrak{CA}_{TM} cl(A) \cup F_R \mathfrak{CA}_{TM} cl(B)$
- (v) $F_R \mathfrak{CA}_{TM} cl(\tilde{0}) = \tilde{0}$
- (vi) $F_R \mathfrak{CA}_{TM} cl(\tilde{1}) = \tilde{1}$

Proof

The proposition is proved from *Definition 3.10 and 3.11*.

Proposition 3.1.5

- (i) Any finite intersection of fuzzy rough algebraic chaotic open is a fuzzy rough algebraic chaotic open.
- (ii) Any finite union of fuzzy rough algebraic chaotic closed is fuzzy rough algebraic chaotic closed.

Proof

(i) Let $\{B_j\}_{j=1}^n$ be a finite collection of fuzzy rough algebraic chaotic open. Then for each j, $F_R \mathfrak{CA}_{TM} int(B) = F_R \mathfrak{CA}_{TM} int(F_R \mathfrak{CA}_{TM} cl(B)).$

Now, by hypothesis and *Proposition 3.1.2*.

$$F_{R}\mathfrak{CA}_{TM}int\left(\bigcap_{j=1}^{n}B_{j}\right) = \bigcap_{j=1}^{n}F_{R}\mathfrak{CA}_{TM}int(B_{j})$$

$$= \bigcap_{j=1}^{n}F_{R}\mathfrak{CA}_{TM}int\left(F_{R}\mathfrak{CA}_{TM}cl(B_{j})\right)$$

$$= F_{R}\mathfrak{CA}_{TM}int\left(\bigcap_{j=1}^{n}\left(F_{R}\mathfrak{CA}_{TM}cl(B_{j})\right)\right)$$

$$\supseteq F_{R}\mathfrak{CA}_{TM}int\left(F_{R}\mathfrak{CA}_{TM}cl\left(\bigcap_{j=1}^{n}B_{j}\right)\right).$$

On the other hand, $\bigcap_{j=1}^{n} B_j \subseteq F_R \mathfrak{CA}_{TM} cl(\bigcap_{j=1}^{n} B_j)$

Which implies,

$$F_{R}\mathfrak{CA}_{TM}int\left(\bigcap_{j=1}^{n}B_{j}\right)\subseteq F_{R}\mathfrak{CA}_{TM}int\left(F_{R}\mathfrak{CA}_{TM}cl\left(\bigcap_{j=1}^{n}B_{j}\right)\right)$$

Therefore,

$$F_{R}\mathfrak{CA}_{TM}int\left(\bigcap_{j=1}^{n}B_{j}\right)=F_{R}\mathfrak{CA}_{TM}int\left(F_{R}\mathfrak{CA}_{TM}cl\left(\bigcap_{j=1}^{n}B_{j}\right)\right)$$

Hence any finite intersection of fuzzy rough algebraic chaotic open is a fuzzy rough algebraic chaotic open.

(ii) Let $\{D_j\}_{j=1}^n$ be a finite collection of

fuzzy rough algebraic chaotic closed. Then for each j, $F_R \mathfrak{CA}_{TM} cl(A) = F_R \mathfrak{CA}_{TM} cl(\mathcal{F}_{\mathcal{RTM}} int(A))$.

Now, by hypothesis and Proposition 3.1.4.

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$$F_{R}\mathfrak{CA}_{TM}cl(\bigcup_{j=1}^{n}A_{j}) = \bigcup_{j=1}^{n}F_{R}\mathfrak{CA}_{TM}cl(A_{j})$$
$$= \bigcup_{j=1}^{n}F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}int(A_{j}))$$
$$= F_{R}\mathfrak{CA}_{TM}cl(\bigcup_{j=1}^{n}(F_{R}\mathfrak{CA}_{TM}int(A_{j})))$$
$$\subseteq F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}int(\bigcup_{j=1}^{n}A_{j})).$$

On the other hand,

$$F_R \mathfrak{CA}_{TM} int \left(\bigcup_{j=1}^n A_j \right) \subseteq \bigcup_{j=1}^n A_j$$

Which implies,

$$F_R \mathfrak{CA}_{TM} cl\left(F_R \mathfrak{CA}_{TM} int\left(\bigcup_{j=1}^n A_j\right)\right) \subseteq F_R \mathfrak{CA}_{TM} cl\left(\bigcup_{j=1}^n A_j\right).$$

Therefore, $F_R \mathfrak{CA}_{TM} cl(\bigcup_{j=1}^n A_j) = F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} int(\bigcup_{j=1}^n A_j))$. Hence any finite rough algebraic closed is a fuzzy rough algebraic closed.

Definition 3.12

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Then (X, \mathfrak{CA}_{TM}) is called a fuzzy rough algebraic regular chaos system if the fuzzy rough chaotic closure of every fuzzy rough chaotic TM algebra is a fuzzy rough chaotic TM algebra.

Proposition 3.1.6

Let (X, \mathfrak{CA}_{TM}) be a fuzzy rough algebraic chaos system. Then the following statements are equivalent:

- (i) (X, \mathfrak{CA}_{TM}) is a fuzzy rough algebraic regular chaos system.
- (ii) For each fuzzy rough algebraic chaotic closed D, $F_R \mathfrak{CA}_{TM} int(D)$ is a fuzzy rough algebraic chaotic closed.
- (iii) For each fuzzy rough algebraic chaotic open B,

 $F_R \mathfrak{CA}_{TM} cl(B) \cup F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} cl(B)') = \tilde{1}.$

(iv) For every pair of fuzzy rough algebraic chaotic open S and R with $F_R \mathfrak{CA}_{TM} cl(A) \cup B = \tilde{1}$

 $F_R \mathfrak{CA}_{TM} cl(A) \cup F_R \mathfrak{CA}_{TM} cl(B) = \tilde{1}.$

Proof

(i) \Rightarrow (ii)

Let *D* be any fuzzy rough algebraic chaotic closed in *X*. Then D' is a fuzzy rough algebraic chaotic open.

Hence, $F_R \mathfrak{CA}_{TM} cl(D)' = (F_R \mathfrak{CA}_{TM} int(D))'$.

Then by (i) $F_R \mathfrak{CA}_{TM} cl(D)'$ is a fuzzy rough algebraic chaotic open. Then $F_R \mathfrak{CA}_{TM} int(D)$ is fuzzy rough algebraic chaotic closed.

(ii) \Rightarrow (iii)

Let B be a fuzzy rough algebraic chaotic open. Then,

$$F_{R}\mathfrak{CA}_{TM}cl(B) \cup F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}cl(B))$$

$$= F_{R}\mathfrak{CA}_{TM}cl(B) \cup F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}int(B)')$$
(1)

Since *B* is a fuzzy rough algebraic chaotic open, *B'* is a fuzzy rough algebraic chaotic closed. Hence by (ii), $F_R \mathfrak{CA}_{TM} int(B)'$ is a fuzzy rough algebraic chaotic closed. Then by *Equation (1)*,

$$F_{R}\mathfrak{CA}_{TM}cl(B) \cup F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}cl(B))'$$

$$= F_{R}\mathfrak{CA}_{TM}cl(B) \cup F_{R}\mathfrak{CA}_{TM}cl(F_{R}\mathfrak{CA}_{TM}int(B)')$$

$$= F_{R}\mathfrak{CA}_{TM}cl(B) \cup F_{R}\mathfrak{CA}_{TM}int(B)'$$

$$= F_{R}\mathfrak{CA}_{TM}cl(B) \cup (F_{R}\mathfrak{CA}_{TM}cl(B))'$$

$$= \tilde{1}.$$

Hence $F_R \mathfrak{CA}_{TM} cl(B) \cup F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} cl(B))' = \tilde{1}.$ (iii) \Rightarrow (iv)

Let S and R fuzzy rough algebraic chaotic open with

$$F_R \mathfrak{CA}_{TM} cl(S) \cup R = \tilde{1}.$$
(2)

By (iii), $\tilde{1} = F_R \mathfrak{CA}_{TM} cl(S) \cup F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} cl(S))'$ $= F_R \mathfrak{CA}_{TM} cl(S) \cup F_R \mathfrak{CA}_{TM} cl(F_R \mathfrak{CA}_{TM} cl(R))'$ $= F_R \mathfrak{CA}_{TM} cl(S) \cup F_R \mathfrak{CA}_{TM} cl(R).$ Hence, $F_R \mathfrak{CA}_{TM} cl(S) \cup F_R \mathfrak{CA}_{TM} cl(R) = \tilde{1}.$

Hence, $F_R \mathfrak{GA}_{TM} cl(S) \cup F_R \mathfrak{GA}_{TM} cl(R) =$ (iv) \Rightarrow (i)

Let S be a fuzzy rough algebraic chaotic open.

Take $R = (F_R \mathfrak{CA}_{TM} cl(S))'$. Then $F_R \mathfrak{CA}_{TM} cl(S) \cup R = \tilde{1}$.

This implies that $F_R \mathfrak{CA}_{TM} cl(S)$ is a fuzzy rough algebraic chaotic open and so (X, \mathfrak{CA}_{TM}) is a fuzzy rough algebraic regular chaos system.

Conclusion:

This work would like to make the upbeat prediction that the proposed definitions and findings of this work can be similarly applied to additional TM-ideals in different structure space. This study will be useful in generating a big impression on prospective researchers, pique their interest in this area and related ones, and broaden their frontiers of knowledge.

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