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FIXED POINT RESULTS FOR THREE MAPPINGS USING CONTRACTIVE CONDITIONS IN TVS-VALUED CONE METRIC SPACE

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ABSTRACT

In this article, we prove fixed point theorems using c-distances for three mappings in tvs-valued cone metric space under contractive conditions. The results obtained in this paper generalize and extend the results presented in the literature. Some examples are presented to validate our obtained results.

Keywords: c-distance, common fixed point, tvs cone-metric space, contraction mapping.

MSC: 47H09; 47H10; 54H25.

1. Introduction

Fixed Point Theory is one of the most effective tools in several branches of mathematics which has enormous application inside as well as outside mathematics. In this field, Banach established the first important result for contraction mapping in complete metric space.

Frechet [2], presented the notion of metric spaces. Following that, Kurepa [19], introduced a more theoretical type of metric space in which ordered vector space is used as a co-domain of a metric in place of the set of real numbers. In written texts, the metric spaces based on vector-valued metric are studied under different names: Pseudo metric spaces, rectangular metric spaces, probabilistic metric spaces, D-metric spaces, cone-valued metric spaces, cone metric spaces (CMS) etc.

Huang and Zhang [3] reintroduced such spaces under the name of cone metric space by replacing a set of real numbers with an ordered Banach space. They likewise then proved some fixed point results for cone metric spaces. Beg et al. [12] and Du [9] generalized CMS to tvs-valued CMS and proved the existence of fixed points for a few couples of mappings with contractive conditions without assuming any normalities for the cone in a tvs-valued CMS [4, 8, 10, 11, 16] where co-domain is an ordered tvs. On the contrary Kada et al. [17] proved a fixed point theorem in CMS using the idea of w-distance. Cho et al. [5] introduced the concept of c-distance which is a cone version of the w-distance. Later on different authors have studied this concept in metric space as well as CMS [6, 7, 8, 10].

To sum up, Beg et al.[11] used tvs-valued CMS and got fixed points which are the outcomes of the results in [10] and [12]. Azam and Rhoades [20] acquired a coincidence fixed point for a pair of mappings satisfying some generalized contractive conditions in tvs-valued CMS. Furthermore, Radenovic et al.[13] gave the notion of tvs-valued CMS with non normal cones.

This study finds the common fixed point for three mappings satisfying a generalized contractive type condition with c-distance in tvs-valued CMS. Our results generalize some significant results of Dubey et al.[6, 20] and Dordevic et.al.[18] and the outcomes referred to in that. It is worth mentioning that our results do not require the assumption that the cone is normal. Example has been given for all positive real numbers to acknowledge these results from the recognized ones.

2. Preliminaries

The following definitions and results will be needed in the sequel.

Definition 2.1: Let E be the real Banach space and the zero element of E be θ. A subset P_p of E is a cone if and only if:

(a) P_p is non empty, closed and $P_p \neq \{\theta\}$;

(b) If a, b are non -negative real numbers with $a, b \geq 0$ and , $s, t \in P_p$ then $as + bt \in P_p$;

(c) If $s \in P_p$, $-s \in P_p$ implies $s \in \theta$.

Given $P_p \subseteq E$, a partial ordering $\preceq$ is defined with respect to P_p as $s \preceq t$ if and only if $(t-s) \in int\, P_p$.

We denote $s < t$ we aim to say that $s \preceq t$ but $s \neq t$ despite $s \ll t$ stands for $(t-s) \in int\, P_p$. Then (E, P_p) is an ordered tvs.

Definition 2.2: If there exists a number k such that $\theta \preceq s \preceq t$ implies that $||s|| \preceq k||t||$ for all $,t \in E$, then the normal constant is the lowest positive integer. and the cone P_p is called normal.

Definition 2.3: Suppose X is a non empty set, $(E,\ P_p)$ an ordered tvs. A mapping or function $d : X \times X \rightarrow E$ is defined such that:

(a) $d(s,t) \geq \theta$ where all $s, t \in X$, $d(s,t) = \theta$ if and only if $s = t$;

(b) $d(s,t) = d(t,s)$ where all $s,t \in X$;

(c) $d(s,w) \preceq d(s,t) + d(t,w)$ where all $s,t,w \in X$. Then (X,d) is called a tvs-valued CMS and the function d is a tvs-cone metric.

In continuation, E is a tvs, θ is a vector (zero vector), P_p is a solid cone, $\preceq$ a partial ordering.

Example 2.4: Suppose $E = R^2\ and\ P_p = \{(a,b) \in E : a,b \geqslant 0\} \subset R^2, X = R^2$ and let us suppose that $d : X \times X \rightarrow E$ is defined by:

$d(a,b) = d\big((a_1,a_2),(b_1,b_1)\big) = (|a_1 - b_1| + |a_2 - b_2|),\ \alpha\, max(|a_1 - b_1|, |a_2 - b_2|)$ in which constant $\alpha \geq 0$. Then (X,d) converts to a CMS over $(E,\ P_p)$. Also, we have P_p is a normal solid cone in which the normal constant k is equal to 1.

Definition 2.5[1]: Let (X, d) is a tvs-valued CMS, $s \in X$ and let $\{s_{m_1}\} \in X$. Then

(a) $\{s_{m_1}\} \to s$ for all $c_1 \in E$ with $\theta \ll c_1$ a natural number m_0 exists, then $d(s_{m_1}, s) \ll c_1$, for all $m_1 \geq m_0$. It is given by $\lim_{m_1 \to \infty} s_{m_1} = s$.

(b) $\{s_{m_1}\}$ is tvs cone Cauchy sequence if for each $c_1 \in E$ with $\theta \ll c_1$ a natural number m_0 exists, such that $d(s_{m_1}, s_{n_1}) \ll c_1$ for all $m_1, n_1 \geq m_0$.

(c) If each Cauchy sequence in X is convergent, then the pair (X, d) is complete.

Now we give the concept of c-distance on a tvs-valued CMS (X, d) Cho et al. in [5].

Definition 2.6[7]: Let (X, d) be a tvs-valued CMS. A function $q : X \times X \to E$ is said to be a c-distance in X under the following circumstances:

(a) $q(s, t) \geq \theta$ for all $s, t \in X$.

(b) $q(s, w) \leq q(s, t) + q(t, w)$ for all $s, t, w \in X$.

(c) $If \{t_{m_1}\} \in X \to t$ where $t \in X$, then for each $m_1 \in N\, q(s, t) \leq w$ for some $s \in X,\ w = w\,x \in P_p$,

(d) For every $c_1 \in E, \theta \ll c_1$ there exists $e \in E$ with $\theta \leq e$ in such a way that $q(w, s) \ll e$ together with $q(w, t) \ll e$ which gives $d(s, t) \ll c_1$.

The lemma given below is the tvs-cone metric version of lemmas from [3, 14].

Lemma 2.7 [3, 14]: Let (X, d) be a tvs-valued CMS in addition to q as the c-distance in X, and two sequences in X are $\{s_{m_1}\}$ and $\{t_{m_1}\}$ where $s, t, w \in X$. Let $\{u_{m_1}\}$ and $\{v_{m_1}\}$ are c-sequences in P_p. Then following conditions hold:

(a) For each $m_1 \in N$, $q(s_{m_1}, t) \leq u_{m_1}$ and $q(s_{m_1}, w) \leq v_{m_1}$ then $t = w$. In general $q(s, t) = \theta$ and $q(s, w) = \theta$ then $t = w$.

(b) For each $m_1 \in N, q(s_{m_1}, t_{m_1}) \leq u_{m_1}$ and $q(s_{m_1}, w) \leq v_{m_1}$ then $t_{m_1} \to w$.

(c) For $n_1 > m_1 > m_1$, $q(s_{m_1}, s_{n_1}) \leq u_{m_1}$ then $\{s_{m_1}\}$ is a Cauchy sequence.

(d) For each $m_1 \in N$, $q(t, s_{m_1}) \leq u_{m_1}$ then $\{s_{m_1}\}$ is a Cauchy sequence.

3. Main Results

In the course of the segment, we establish a unique common fixed point result for three mappings using generalized type contractive conditions.

Theorem 3.1: Let $f, g, h : X \to X$ be three continuous and sub-sequentially convergent mappings in a complete tvs-valued CMS (X, d), where P_p be a solid cone and q be the c-distance, such that $f \cup g \subseteq h$. Let A and B are two nonnegative real numbers for all $s, t \in X$ so that these conditions can be satisfied:

(a) $A + 2B < 1$.

(b) $q(fs, gt) \leq Aq(hs, ht) + B\,[q(fs, ht) + q(gt, hs)]$ and

(c) $q(gt, fs) \leq Aq(ht, hs) + B\,[q(ht, fs) + q(hs, gt)]$ for all $s, t \in X$.

Then the map h has a unique common fixed point $s* \in X$ and for any $s \in X$, the iterative sequence hs_{m_1}converges to a fixed point. If $\omega = h\omega$, then $q(f\omega, g\omega) = \theta$.

Let $s_{2m_1+1} = fs_{2m_1} = hs_{2m_1+1}$ and $s_{2m_1+2} = gs_{2m_1+1} = hs_{2m_1+2}$.

Then we have

$$q(s_{2m_1+1}, s_{2m_1+2}) \leq q(fs_{2m_1}, gs_{2m_1+1})$$
$$= Aq(hs_{2m_1}, hs_{2m_1+1}) + B[q(fs_{2m_1}, hs_{2m_1+1}) +$$
$$+q(gs_{2m_1+1}, hs_{2m_1})]$$
$$\leq Aq(s_{2m_1}, s_{2m_1+1}) + B[q(s_{2m_1+1}, s_{2m_1+1}) +$$
$$+q(s_{2m_1+2}, s_{2m_1}]$$
$$\leq Aq(s_{2m_1}, s_{2m_1+1}) + B[q(s_{2m_1}, s_{2m_1+1}) +$$
$$+q(s_{2m_1+1}, s_{2m_1+2})],$$
$$(1-B)q(s_{2m_1+1}, s_{2m_1+2}) \leq Aq(s_{2m_1}, s_{2m_1+1}) + B\, q(s_{2m_1}, s_{2m_1+1}),$$
$$\leq (A+B)q(s_{2m_1}, s_{2m_1+1})$$
$$\leq \frac{A+B}{1-B} q(s_{2m_1}, s_{2m_1+1})$$
$$\leq kq(s_{2m_1}, s_{2m_1+1})$$
$$\leq k^2 q(s_{2m_1-1}, s_{m_1})$$
$$\leq k^{m_1} q(s_0, s_1). \dots\dots\dots\dots\dots\dots (1)$$

$$Where\ k = \frac{A+B}{1-B} < 1.$$

Similarly

$$q(s_{2m_1+2}, s_{2m_1+1}) \leq q(gs_{2m_1+1}, fs_{m_1})$$
$$= Aq(hs_{2m_1+1}, hs_{2m_1}) + B[q(hs_{2m_1+1}, fs_{2m_1}) + q(hs_{2m_1}, gs_{2m_1+1})]$$
$$\leq Aq(s_{2m_1+1}, s_{2m_1}) + B[q(s_{2m_1+1}, s_{2m_1+1}) + q(s_{2m_1+2}, s_{2m_1})]$$
$$\leq Aq(s_{2m_1}, s_{2m_1+1}) + B[q(s_{2m_1}, s_{2m_1+1}) + q(s_{2m_1+1}, s_{2m_1+2})]$$
$$(1-B)q(s_{2m_1+2}, s_{2m_1+1}) \leq (A+B)q(s_{2m_1+1}, s_{2m_1}),$$
$$\leq \frac{A+B}{1-B} q(s_{2m_1+1}, s_{2m_1})$$
$$\leq kq(s_{2m_1+1}, s_{2m_1})$$
$$\leq k^2 q(s_{2m_1}, s_{2m_1-1})$$
$$\leq k^{m_1} q(x_1, x_0). \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

$$Where\ k = \frac{A+B}{1-B} < 1.$$

Let us denote $\omega_{m_1} = (A+B)\omega_{m_1-1} + (1-B)\omega_{m_1}$.

$i.e\ \omega \leq k\omega_{m_1-1}$ with $0 \leq k = \frac{A+B}{1-B} < 1$.

since $A + 2B < 1$,

By induction, $\omega_{m_1} \leq k^{m_1}\omega_0$ and from (1) and (2) we get

$q(s_{2m_1+1}, s_{2m_1+2}) \leq \omega_{m_1} \leq \frac{k^{m_1}}{1-k} [q\,(s_1, s_0) + q(s_0, s_1)].$

which gives

$q(s_{m_1}, s_{n_1}) \leq k^{m_1}[q(s_1, s_0) + q(s_0, s_1)]\, for\ all\ m_1 \geq 1.$

Then $s_{m_1}$is Cauchy. Moreover, if X is complete, then s_{m_1} converges to a point $s^* \in X$ such that $s_{m_1} \to s^*$ as $m_1 \to \infty$.

Since f, g, h are sub-sequentially convergent and continuous, and $f \cup g \subseteq h$ it easily follows that $fs^* = gs^* = hs^* = s^*$.

Thus the mapping h has a common fixed point. Let $\omega \in X$ satisfying $f = g\omega = h\omega = \omega$ then from (1) and (2)

$q(\omega\,, \omega) = q(f\omega\,, g\omega) \leq Aq(\omega\,, \omega) + B\,[q(\omega\,, \omega) + q(\omega\,, \omega)]$

$q(\omega\,, \omega) = q(f\omega\,, g\omega) = (A + 2B)\, q\,(\omega\,, \omega)$

$As\ A + 2B < 1$, it gives that $q(\omega\,, \omega) = \theta$.

Corollary 3.2: Let $f, g, h : X \to X$ be three continuous and sub-sequentially convergent mappings in a complete tvs-valued CMS (X, d), where P_p be a solid cone and q be the c-distance, in such a way that $f \cup g \subseteq h$. Let A and B are two non-negative real numbers for all $s, t \in X$ so that these conditions can be satisfied:

(a) $A + 2B < 1$.

(b) $q(fs, gt) \leq Aq(hs, ht) + B\,[q(fs, hs) + q(gt, ht)]$ and

(c) $q(gt, fs) \leq Aq(ht, hs) + B\,[q(hs, fs) + q(ht, gt)]$ for all $s, t \in X$.

Then the map h has a unique common fixed point $s^* \in X$ and for all $s \in X$, the iterative sequence hx_{m_1} converges to a fixed point. If $\omega = h\omega$, then $q\,(f\omega, g\omega) = \theta$.

Theorem 3.3: Let $f, g, h : X \to X$ be three continuous and sub-sequentially convergent mappings in a complete tvs-valued CMS (X, d), P_p be a solid cone and q be the c-distance, in such a way that $f \cup g \subseteq h$. Let A, B and C are three non- negative real numbers for all $s, t \in X$ so that these conditions can be satisfied:

(a) $A + 2B + 2C < 1$.

(b) $q\,(fs, gt) \leq Aq\,(hs, ht) + B\,[q\,(ht, fs) + q\,(hs, gt)] + C\,[q\,(hs, fs) + q\,(ht, ft)]$ and

(c) $q(gt, fs) \leq Aq(ht, hs) + B\,[q(fs, ht) + q(gt, hs)] + C\,[q(fs, hs) + q(ft, ht)]\, for\ all\ s, t \in X.$

Then the map h has a unique common fixed point $s^* \in X$ and for any $s \in X$, the iterative sequence hs_{m_1} converges to a fixed point. If $\omega = h\omega$, then $q(f\omega\,, g\omega) = \theta$.

Proof: Let $s_{2m_1+1} = fs_{2m_1} = hs_{2m_1+1}\ and\ s_{2m_1+2} = gs_{2m_1+1} = hs_{2m_1+2}$.

Then

$$q\,(s_{2m_1+1}, s_{2m_1+2}) \leq q\,(fs_{2m_1}, gs_{2m_1+1})$$

$$= Aq\left(hs_{2m_1}, hs_{2m_1+1}\right) + B\left[q\left(hs_{2m_1+1}, fs_{2m_1}\right) + q\left(hs_{2m_1}, gs_{2m_1+1}\right)\right]$$

$$+C\,[q\,(hs_{2m_1}, fs_{2m_1}) + q\,(hs_{2m_1+1}, gs_{2m_1+1})]$$

$$\leq Aq\left(s_{2m_1}, s_{2m_1+1}\right) + B\left[q\left(s_{2m_1+1}, s_{2m_1+1}\right) + q\left(s_{2m_1}, s_{2m_1+1}\right)\right]$$

$$+C\left[q\left(s_{2m_1}, s_{2m_1+1}\right) + q\left(s_{2m_1+1}, s_{2m_1+2}\right)\right]$$

$$\leq Aq\left(s_{2m_1}, s_{2m_1+1}\right) + B\left[q(s_{2m_1+1}, s_{2m_1+1}) + q\left(s_{2m_1}, s_{2m_1+1}\right)\right]$$

$$+C\left[q(s_{2m_1}, s_{2m_1+1}) + q(s_{2m_1+1}, s_{2m_1+2})\right],$$

$$(1-B-C)q\left(s_{2m_1+1}, s_{2m_1+2}\right) Aq\left(s_{2m_1}, s_{2m_1+1}\right) + B\,q\left(s_{2m_1}, s_{2m_1+1}\right) + C\,q\left(s_{2m_1}, s_{2m_1+1}\right)$$

$$\leq (A + B + C)q\left(s_{2m_1}, s_{2m_1+1}\right).$$

And hence

$$q\left(s_{2m_1+1}, s_{2m_1+2}\right) \leq \frac{(A+B+C)}{(1-B-C)} q\left(s_{2m_1}, s_{2m_1+1}\right)$$

$$\leq kq\left(s_{2m_1}, s_{2m_1+1}\right)$$

$$\leq k^2 q\left(s_{2m_1}, s_{2m_1}\right)$$

$$\leq k^{m_1} q\left(s_0, s_1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Where $k = \frac{(A+B+C)}{(1-B-C)} < 1.$

Similarly

$$q\left(s_{2m_1+2}, s_{2m_1+1}\right) \leq q\left(gs_{2m_1+1}, fs_{2m_1}\right)$$

$$= Aq(hs_{2m_1}, hs_{2m_1+1}) + B\left[q(fs_{2m_1}, hs_{2m_1+1}) + q(gs_{2m_1+1}, hs_{2m_1})\right] +$$

$$C\left[q(fs_{2m_1}, hs_{2m_1}) + q(gs_{2m_1+1}, hs_{2m_1+1})\right]$$

$$\leq Aq\left(s_{2m_1}, s_{2m_1+1}\right) + B\left[q\left(s_{2m_1+1}, s_{2m_1+1}\right) + q\left(s_{2m_1+1}, s_{2m_1}\right)\right] +$$

$$C\left[q(s_{2m_1+1}, s_{2m_1}) + q(s_{2m_1+2}, s_{2m_1+1})\right]$$

$$\leq Aq(s_{2m_1+1}, s_{2m_1}) + B\left[q(s_{2m_1+1}, s_{2m_1}) + q(s_{2m_1+2}, s_{2m_1+1})\right] +$$

$$C\left[q(s_{2m_1+1}, s_{2m_1}) + q\left(s_{2m_1+2}, s_{2m_1+1}\right)\right],$$

$$(1-B-C)q\left(s_{2m_1+2}, s_{2m_1+1}\right) \leq Aq\left(s_{2m_1+1}, s_{2m_1}\right) + B\,q\left(s_{2m_1+1}, s_{2m_1}\right) +$$

$$C\,q\left(s_{2m_1+1}, s_{2m_1}\right)$$

$$\leq (A + B + C)q(s_{2m_1+1}, s_{2m_1})$$

And hence

$$q(s_{2m_1+2}, s_{2m_1+1}) \leq \frac{(A+B+C)}{(1-B-C)}$$

$$q\left(s_{2m_1+1}, s_{2m_1}\right)$$

$$\leq kq\left(s_{2m_1+1}, s_{2m_1}\right)$$

$$\leq k^2 q\left(s_{2m_1}, s_{2m_1-1}\right)$$

$$\leq k^{m_1} q(s_1, s_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where $k = \frac{(A+B+C)}{(1-B-C)} < 1.$

Let us denote $\omega_{m_1} = (A + B + C)\omega_{m_1-1} + (1-B-C)\omega_{m_1}.$

i.e $\omega_{m_1} \leq k\,\omega_{m_1} - 1$with $0 \leq k = \frac{(A+B+C)}{(1-B-C)} < 1$.

since $A + 2B + 2C < 1$,

By induction, $\omega_{m_1} \leq k^{m_1}\omega_0$ and from (3) and (4)

$$q(s_{2m_1+1}, s_{2m_1+2}) \leq \omega_{m_1} \leq \frac{k^{m_1}}{1-k}\,[q(s_1, s_0) + q(s_0, s_1)].$$

which gives

$q(s_{m_1}, s_{m_1}) \leq k^{m_1}\,[q(s_1, s_0) + q(s_0, s_1)]$ for all $m_1 \geq 1$.

Then s_{m_1} is Cauchy. Moreover if X is complete, then s_{m_1} converges to a point, such that $s_{m_1} \to s^*$ as $m_1 \to \infty$.

As f, g, h are sub-sequentially convergent and continuous, and $f \cup g \subseteq h$ it easily follows that $fs^* = gs^* = hs^* = s^*$.

Thus the mapping h has a common fixed point. Let $\omega \in X$ satisfying $f\omega = g\omega = h\omega = \omega$, from (3) and (4) we have

$$q(\omega, \omega) = q(f\omega, g\omega) \leq Aq(\omega, \omega) + B\,[q(\omega, \omega) + q(\omega, \omega)] + C\,[q(\omega, \omega) + q(\omega, \omega)]$$

$$q(\omega, \omega) = q(f\omega, g\omega) = (A + 2B + 2C)\,q(\omega, \omega).$$

As $A + 2B + 2C < 1$, it follows that $q(\omega, \omega) = \theta$.

Example: Let (X, d) be a tvs-valued CMS , $\preceq$ be a partial ordering and $E = R^2$ and $P_p = \{(s,t) \in E : s, t \geqslant 0\} \subset R^2, X = R^2$. Let $d : X \times X \to E$ such that $d(s,t) = s + t$. Define a mapping $q : X \times X \to E$ by $q(s,t) = \frac{1}{2}(s + t)$ for all $s, t \in X$.

Let $f, g, h : X \to X$ by $fs = \frac{2s}{3}$, $gs = \frac{4s}{3}$, and $Ss = 2s$ for all $s \in X$.

Given below are the cases of $s, t \in X$ in detail.

Case I: If $s, t \in [0,1)$, taking $s = \frac{1}{2}, t = \frac{1}{3}$ we have $q(fs, gt) = \frac{1}{2}\left[\frac{2s}{3} + \frac{4t}{3}\right] = 0.38$.

Using the contractive condition found in equation (3.1), we now have

$q(fs, gt) \leq Aq(hs, ht) + B\,[q(fs, ht) + q(gt, hs)]$.

Given A and B are two non negative real numbers, choose $A = \frac{1}{3}$ and $B = \frac{1}{6}$, clearly $A + 2B < 1$.

Now that we have changed the values of s and t, we obtain

$$0.38 \leq A\left[\frac{1}{2}(2s + 2t)\right] + B\left[\frac{1}{2}\left\{\left(\frac{2s}{3} + 2t\right) + \left(\frac{4t}{3} + 2s\right)\right\}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}\left(2 \times \frac{1}{2} + 2 \times \frac{1}{3}\right)\right] + \frac{1}{6}\left[\frac{1}{2}\left\{\left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{4}{9} + 1\right)\right\}\right]$$

$0.38 \leq 0.48$.

Consequently, theorem 3.1's two contractive requirements are satisfied..

Case II: If $s \in [0,1), t \in [1,\infty)$ taking $s = \frac{1}{2}, t = 1$ we have $q(fs, gt) = \frac{1}{2}\left[\frac{2s}{3} + \frac{4t}{3}\right] = 0.83$.

Using the contractive condition found in equation (3.1), we now have

$q(fs, gt) \leq Aq(hs, ht) + B\,[q(fs, ht) + q(gt, hs)]$.

Given A and B are two non negative real numbers, choose $A = \frac{1}{3}$ and $B = \frac{1}{6}$, clearly $A + 2B < 1$.

Now that we have changed the values of s and t, we obtain

$$0.83 \leq A\left[\frac{1}{2}(2s + 2t)\right] + B\left[\frac{1}{2}\left\{\left(\frac{2s}{3} + 2t\right) + \left(\frac{4t}{3} + 2s\right)\right\}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}(1 + 2)\right] + \frac{1}{6}\left[\frac{1}{2}\left\{\left(\frac{1}{3} + 2\right) + \left(\frac{4}{3} + 1\right)\right\}\right]$$

$0.83 \leq 0.88$.

Consequently, theorem 3.1's two contractive requirements are satisfied.

Case III: If $s, t \in [1, \infty)$, taking $s = 1$ and $t = 2$ we have $q(fs, gt) = \frac{1}{2}\left[\frac{2s}{3} + \frac{4t}{3}\right] = 1.66$.

Given A and B are two non negative real numbers, choose $A = \frac{1}{3}$ and $B = \frac{1}{6}$, clearly $A + 2B < 1$.

Now that we have changed the values of s and t, we obtain

$$1.66 \leq A\left[\frac{1}{2}(2s + 2t)\right] + B\left[\frac{1}{2}\left\{\left(\frac{2s}{3} + 2t\right) + \left(\frac{4t}{3} + 2s\right)\right\}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}(2 + 4)\right] + \frac{1}{6}\left[\frac{1}{2}\left\{\left(\frac{2}{3} + 4\right) + \left(\frac{8}{3} + 2\right)\right\}\right]$$

$1.66 \leq 1.77$.

Consequently, theorem 3.1's two contractive requirements are satisfied.

The notations used here are as follows: tvs-valued CMS for topological vector space-valued cone metric space and tvs for topological vector space.

5. Conclusion

In this article, we establish the existence and uniqueness of the common fixed point for three mappings using c-distance in tvs-valued CMS with generalized type contractive conditions. Our results in this paper generalize and improve the results proved by Dubey et al.[6, 20] and Dordevic et al.[18] in the sense that we employ three mappings instead of two with a reduced number of parameters in the conditions and by replacing cone metric space with tvs-valued cone metric space, which further extends the scope of our results. Moreover, the utility of the theorem proved above has been illustrated by providing an example.

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