



A STUDY ON QUASI UNIFORM DYNAMICAL SYSTEM

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ABSTRACT

This paper introduces a new concept called fuzzy rough algebraic quasi uniformity on a fuzzy rough TM dynamical system. It is highlighting the properties of fuzzy rough quasi uniform dynamical system space. This paper proves that any finite intersection of fuzzy rough quasi uniform open algebraic is a fuzzy rough quasi uniform open algebraic and any finite union of fuzzy rough quasi uniform closed algebraic is a fuzzy rough quasi uniform closed algebraic.

Keywords: fuzzy rough algebraic quasi uniformity, fuzzy rough quasi uniform dynamical system, fuzzy rough algebraic uniform dynamical system, fuzzy rough quasi uniform interior, fuzzy rough quasi uniform closure.

1. Introduction

Yong Chan Kim and Jung Mi Ko [3] established the idea of L-fuzzy topologies and L-fuzzy quasi-uniform spaces. D. Vidhya, E. Roja, and M.K. Uma [2] proposed the concept of soft fuzzy disconnectedness and connectedness using soft fuzzy quasi uniform B open sets. A structure (ζ, G, X) with G being a fuzzy topological group, X being a fuzzy rough algebraic TM system, and ζ being a continuous fuzzy rough TM continuous function from $G \times X \rightarrow X$ is a fuzzy rough TM dynamical system.

2. Preliminaries

Definition 2.1 [5]

Fuzzy sets 0_X and 1_X that are both fuzzy open and fuzzy closed are considered to be fuzzy connected sets in a fuzzy topological space (X, T) .

3. QUASI UNIFORM DYNAMICAL SYSTEM

Definition 3.1

A fuzzy rough algebraic continuous mapping \mathbb{F} exists between a fuzzy rough algebraic TM system (X, TM) and a fuzzy rough algebraic TM system (Y, TM) , provided that every fuzzy rough algebraic's inverse image is a member of the fuzzy rough algebraic TM system.

Definition 3.2

Consider G , a fuzzy rough algebraic and X be a fuzzy rough algebraic TM system. If $\zeta: G \times X \rightarrow X$ satisfies the following properties:

- (i) ζ is fuzzy rough algebraic continuous.
- (ii) $\zeta(0, x) = x$
- (iii) $\zeta(s, (t, x)) = \zeta(s + t, x)$

Then (ζ, G, X) is called a fuzzy rough TM dynamical system.

Definition 3.3

A fuzzy rough algebraic quasi uniformity on $X = (X_L, X_U)$ is sub algebra $\mathfrak{U} \subseteq \zeta$, where (ζ, G, X) is fuzzy rough TM dynamical system satisfying the following axioms:

- (i) If $s \in \mathfrak{U}$, $s \subseteq h$ and $h \in \zeta$ then $h \in \mathfrak{U}$.
- (ii) If $s_1, s_2 \in \mathfrak{U}$ then there exists $h \in \mathfrak{U}$ such that $h \subseteq s_1 \cap s_2$.
- (iii) For every $s \in \mathfrak{U}$, there exist $h \in \mathfrak{U}$ such that $h \circ h \subseteq s$.

The pair (X, \mathfrak{U}) is called a fuzzy rough quasi uniform dynamical system.

A fuzzy rough algebraic quasi uniformity on X is called a fuzzy rough uniformity if the following condition holds:

- (iv) $s \in \mathfrak{U}$ implies $s^{-1} \in \mathfrak{U}$, $s^{-1}: X \rightarrow \zeta \times X$ defined by

$$s^{-1}(A) = \cap \{B: s(B)' \subseteq A'\}.$$

The pair (X, \mathfrak{U}) is called fuzzy rough algebraic uniform dynamical system.

Definition 3.4

Let (X, \mathfrak{U}) be a fuzzy rough algebraic uniform dynamical system. The operator $Int_{TM}: \zeta \times X \rightarrow X$ defined by

$$Int_{TM}(A) = \cup \{B \in \zeta(X), s(B) \subseteq A \text{ for some } s \in \mathfrak{U}\}.$$

Definition 3.5

Let (X, \mathfrak{U}) be a fuzzy rough algebraic uniform dynamical system. Then

$$TM_{\mathfrak{U}} = \{A \in \zeta(X): Int_{TM}(A) = A\}$$

Is called the fuzzy rough TM dynamical system generated by \mathfrak{U} . The pair $(X, TM_{\mathfrak{U}})$ is called the fuzzy rough quasi uniform dynamical system space. The members of $TM_{\mathfrak{U}}$ are called the fuzzy rough quasi uniform open algebraic. The complement of a fuzzy rough quasi uniform open algebraic is fuzzy rough quasi uniform closed algebraic.

Definition 3.6

Let $(X, TM_{\mathfrak{U}})$ be a fuzzy rough quasi uniform dynamical system space. Then the fuzzy rough quasi uniform interior (in short $\mathcal{F}_{RTM_{\mathfrak{U}}}Qint$) and the fuzzy rough quasi uniform closure (in short $\mathcal{F}_{RTM_{\mathfrak{U}}}Qcl$) of a fuzzy rough algebraic A in $(X, TM_{\mathfrak{U}})$ is defined as follows:

$$\begin{aligned} &\mathcal{F}_{RTM_{\mathfrak{U}}}Qint(A) \\ &= \cup \{B: B \subseteq A \text{ and } B \text{ is a fuzzy rough quasi uniform open algebraic} \} \end{aligned}$$

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)$$

$$= \cap \{B: B \supseteq A \text{ and } B \text{ is a fuzzy rough quasi uniform closed algebraic}\}.$$

Proposition 3.1

Let $(X, TM_{\mathcal{U}})$ be a fuzzy rough quasi uniform dynamical system space. Let $A = (A_L, A_U)$ fuzzy rough quasi uniform algebraic. Then the following conditions hold:

- (i) $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)$ is a fuzzy rough quasi uniform open algebraic.
- (ii) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)$ is a fuzzy rough quasi uniform closed algebraic.
- (iii) A is a fuzzy rough quasi uniform open algebraic if and only if $A = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)$.

Proof

The proof follows from the *Definition 3.6*

Proposition 3.2

Let $(X, TM_{\mathcal{U}})$ be a fuzzy rough quasi uniform dynamical system space. Then for any fuzzy rough quasi uniform algebraic A in $(X, TM_{\mathcal{U}})$, the following properties holds:

- (i) $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A) \subseteq A \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)$.
- (ii) $(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A))' = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)'$
- (iii) $(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)'$

Proof

The proof follows from the *Definition 3.5*.

Proposition 3.3

Let $\{B_j\}_{j \in J}$ be a family of fuzzy rough quasi uniform algebraic in $(X, TM_{\mathcal{U}})$, and J be an indexed set.

Then for $j \in J$,

- (i) $\cup_j \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B_j) \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\cup_j (B_j))$
- (ii) $\cup_j \mathcal{F}_{RTM_{\mathcal{U}}}Qint(B_j) \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\cup_j (B_j))$

Also, for any finite $\in J$, $\cup_n \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B_n) = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\cup_n (B_n))$.

Proof

The proof is obvious.

Proposition 3.4

Let $(X, TM_{\mathcal{U}})$ be a fuzzy rough quasi uniform dynamical system space. Let C and D be any two fuzzy rough quasi uniform algebraic. Then the following conditions hold:

- (i) $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C) \subseteq C$.
- (ii) $C \subseteq D \Rightarrow \mathcal{F}_{RTM_{\mathcal{U}}}Qint(C) \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qint(D)$
- (iii) $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C)) = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(C)$
- (iv) $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C \cap D) = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(C) \cap \mathcal{F}_{RTM_{\mathcal{U}}}Qint(D)$

$$(v) \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\tilde{0}) = \tilde{0}$$

$$(vi) \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\tilde{1}) = \tilde{1}$$

Proof

By *definition 3.6* the proof follows

Proposition 3.5

Let $(X, TM_{\mathcal{U}})$ be a fuzzy rough quasi uniform dynamical system space. Let E and S be any two fuzzy rough quasi uniform algebraic. Then the following conditions hold:

- (i) $E \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E)$.
- (ii) $E \subseteq S \Rightarrow \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E) \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(S)$
- (iii) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E)) = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E)$
- (iv) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E \cup S) = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(E) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(S)$
- (v) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\tilde{0}) = \tilde{0}$
- (vi) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\tilde{1}) = \tilde{1}$

Proof

The proof follows from *definition 3.6*.

Proposition 3.6

- (i) Any finite intersection of fuzzy rough quasi uniform open algebraic is a fuzzy rough quasi uniform open algebraic.
- (ii) Any finite union of fuzzy rough quasi uniform closed algebraic is a fuzzy rough quasi uniform closed algebraic.

Proof

- (i) Let $\{C_j\}_{j=1}^n$ be a finite collection of fuzzy rough quasi uniform open algebraic. Then for each j ,

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C) = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(C)).$$

Now, by hypothesis and *Proposition 3.3*.

$$\begin{aligned} \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcap_{j=1}^n C_j) &= \bigcap_{j=1}^n \mathcal{F}_{RTM_{\mathcal{U}}}Qint(C_j) \\ &= \bigcap_{j=1}^n \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(C_j)) \\ &= \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcap_{j=1}^n (\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(C_j))) \\ &\supseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcap_{j=1}^n C_j)). \end{aligned}$$

On the other hand,

$$\bigcap_{j=1}^n C_j \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcap_{j=1}^n C_j)$$

Which implies,

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcap_{j=1}^n C_j) \subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcap_{j=1}^n C_j)).$$

Therefore, $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcap_{j=1}^n C_j) = \mathcal{F}_{RTM_{\mathcal{U}}}Qint(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcap_{j=1}^n C_j))$.

Hence any finite intersection of fuzzy rough quasi uniform open algebraic is a fuzzy rough quasi uniform open algebraic.

- (ii) Let $\{C_j\}_{j=1}^n$ be a finite collection of fuzzy rough quasi uniform closed algebraic. Then for each ,

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(C) = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C)).$$

Now, by hypothesis and *Proposition 3.3*.

$$\begin{aligned} \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcup_{j=1}^n C_j) &= \bigcup_{j=1}^n \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(C_j) \\ &= \bigcup_{j=1}^n \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C_j)) \\ &= \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcup_{j=1}^n (\mathcal{F}_{RTM_{\mathcal{U}}}Qint(C_j))) \\ &\subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcup_{j=1}^n C_j)). \end{aligned}$$

On the other hand,

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcup_{j=1}^n C_j) \subseteq \bigcup_{j=1}^n C_j$$

which implies, $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcup_{j=1}^n C_j))$

$$\subseteq \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcup_{j=1}^n C_j).$$

Therefore, $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\bigcup_{j=1}^n A_j) = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(\bigcup_{j=1}^n A_j))$. Hence any finite fuzzy rough quasi uniform closed algebraic is a fuzzy rough quasi uniform closed algebraic.

Definition 3.7

Let (X, \mathcal{U}) be a fuzzy rough algebraic uniform dynamical system. Then (X, \mathcal{U}) is said to be a fuzzy rough algebraic basic uniform dynamical system if fuzzy rough quasi uniform closure of every fuzzy rough algebraic quasi uniformity is a fuzzy rough algebraic quasi uniformity.

Proposition 3.7

Let (X, \mathcal{U}) be a fuzzy rough algebraic uniform dynamical system. Then the following statements are equivalent:

- (i) (X, \mathcal{U}) is a fuzzy rough algebraic basic uniform dynamical system.
- (ii) For any fuzzy rough quasi uniform closed algebraic A , $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)$ is a fuzzy rough quasi uniform closed algebraic.
- (iii) For each fuzzy rough quasi uniform open algebraic A ,

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)') = \tilde{1}.$$

- (iv) For each fuzzy rough quasi uniform open algebraic A and B with $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup B = \tilde{1}$,
 $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B) = \tilde{1}$.

Proof

(i) \Rightarrow (ii)

Let A be any fuzzy rough quasi uniform closed algebraic in X . Then A' is a fuzzy rough quasi uniform open algebraic.

Now, $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)' = (\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A))'$.

Then by (i) $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)'$ is a fuzzy rough quasi uniform open algebraic. Then $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)$ is fuzzy rough quasi uniform closed algebraic.

(ii) \Rightarrow (iii)

Let A be a fuzzy rough quasi uniform open algebraic. Then,

$$\begin{aligned} \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' \\ = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)') \end{aligned} \quad (1)$$

Since A is a fuzzy rough quasi uniform open algebraic. Now A' is a fuzzy rough quasi uniform closed algebraic. Hence by (ii), $\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)'$ is a fuzzy rough quasi uniform closed algebraic.

Then by Equation (1),

$$\begin{aligned} \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' \\ = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)') \\ = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qint(A)' \\ = \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup (\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' \\ = \tilde{1}. \end{aligned}$$

Hence $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' = \tilde{1}$.

(iii) \Rightarrow (iv)

Let A and B be a fuzzy rough quasi uniform open algebraic with

$$\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup B = \tilde{1}. \quad (2)$$

$$\begin{aligned} \text{By (iii), } \tilde{1} &= \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))' \\ &= \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B))' \\ &= \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B). \end{aligned}$$

Hence, $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup \mathcal{F}_{RTM_{\mathcal{U}}}Qcl(B) = \tilde{1}$.

(iv) \Rightarrow (i)

Let A be a fuzzy rough quasi uniform open algebraic.

Put $B = (\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A))'$. Then $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A) \cup B = \tilde{1}$.

This implies that $\mathcal{F}_{RTM_{\mathcal{U}}}Qcl(A)$ is a fuzzy rough quasi uniform open algebraic and so (X, \mathcal{U}) is a fuzzy rough algebraic basic uniform dynamical system.

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Cite this Article:

M. Srividya, D. Vidhya G. Jayalalitha "A Study on Quasi Uniform Dynamical System", International Journal of Scientific Research in Modern Science and Technology (IJSRMST), ISSN: 2583-7605 (Online), Volume 2, Issue 12, pp. 31-37, December 2023. **Journal URL:** <https://ijrmst.com/>