

SOME SPECIAL R-GENERALIZED RECURRENT FINSLER SPACE**P. C. SRIVASTAVA**

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ABSTRACT:

In this paper we have taken into account Cartan's curvature tensor R_{jkh}^i and have defined R-generalized recurrent and special R-generalized recurrent Finsler spaces of the order two. After these observations we find result in the form of theorems.

Keywords: R- generalized recurrent Finsler spaces, Cartan's curvature tensor, special R-generalized recurrent Finsler spaces, Ricci tensor, h - curvature tensor, $h\nu$ - curvature tensor and ν - curvature tensor.

1. INTRODUCTION:

The properties of recurrent Riemannian space by assuming different types of curvature tensors in V_n are introduced by Matsumoto [1], Adati and Miyazawa [5], Walker [7]. Roy [3] has defined genralised 2-recurrent in Riemannian space. An attempt has been made by Pandey and Khan [4] to extend the theory of Finsler geometry to the generalized 2- recurrent curvature tensor. Mishra [6] has presented different types of recurrent Finsler space using Barwald & Cartan curvature tensors.

2. R-GENERALIZED RECURRENT AND SPECIAL R-GENERALIZED RECURRENT FINSLER SPACE :

In a Finsler space, the Cartan curvature tensor R_{jkh}^i is exists recurrence properties w.r. to the requisite connection (Verma [8]). Therefore, the characterized property

$$(1.1) R_{jkh|m}^i = \mu_m R_{jkh}^i ,$$

Here, $\mu_m(x)$ is known as non-zero recurrence vector field. An R- recurrent space of order two is a Finsler space in which the curvature tensor exits under assumption

$$(1.2) R_{jkh|m|l}^i = \alpha_{lm} R_{jkh}^i ,$$

where $\alpha_{lm}(x, \dot{x})$ is associated tensor of recurrence and it is covariant of order two.

We now take into consideration a Finsler space in which the curvature under assumption satisfies

$$(1.3) R_{jkh|m|l}^i = \mu_l R_{jkh|m}^i + \alpha_{lm} R_{jkh}^i$$

and

$$(1.4) R_{jkh|m|l}^i = \mu_m R_{jkh|l}^i + \alpha_{lm} R_{jkh}^i .$$

Here, we shall call the space satisfying (1.3) and (1.4) as generalized R- recurrent Finsler space of the second order of first and second kinds respectively. In this continuation it has also been observed that if the curvature tensor under assumption satisfies

$$(1.5) R_{jkh|m|l}^i = \mu_l R_{jkh|m}^i$$

and

$$(1.6) R_{jkh|m|l}^i = \mu_m R_{jkh|l}^i ,$$

then we call such a space as special R- generalized recurrent Finsler space of the second order of first and second kinds respectively. If we further suppose that the recurrence vector appearing in (1.3) and (1.4) is zero then the space reduces into a R- recurrent Finsler space of the second order. We now transvect the equations (1.3) to (1.6) by covariant fundamental tensor g_{ip} and thereafter, we get

$$(1.7) \quad R_{pjkh|m|l} = \mu_l R_{pjkh|m} + \alpha_{lm} R_{pjkh} ,$$

$$(1.8) \quad R_{pjkh|m|l} = \mu_m R_{pjkh|l} + \alpha_{lm} R_{pjkh} ,$$

$$(1.9) \quad R_{pjkh|m|l} = \mu_l R_{pjkh|m} ,$$

and $(1.10) \quad R_{pjkh|m|l} = \mu_m R_{pjkh|l} ,$

whereas, if we transvect the equations (1.7) to (1.10) by contravariant fundamental tensor g^{ip} immediately we get back the set of equations (1.3) to (1.6). So we can state the followings:

THEOREM (1):

The characterizing properties of R-generalized and special R-generalized recurrent Finsler space of the second order of the two kinds are respectively given by the equation (1.7) to (1.10).

THEOREM (2):

The equations given by (1.3) to (1.6) and (1.7) to (1.10) are respectively found to be equivalent in R- generalized and special R- generalized recurrent Finsler spaces of the second order of the first and second kinds.

Now allowing a contraction in the set of equations (1.3) to (1.6) w.r.t. the indices i & h , we have

$$(1.11) \quad R_{jk|m|l} = \mu_l R_{jk|m} + \alpha_{lm} R_{jk} ,$$

$$(1.12) \quad R_{jk|m|l} = \mu_m R_{jk|l} + \alpha_{lm} R_{jk} ,$$

$$(1.13) \quad R_{jk|m|l} = \mu_l R_{jk|m} ,$$

and $(1.14) \quad R_{jk|m|l} = \mu_m R_{jk|l} ,$

Respectively. After these observations, therefore we can state:

THEOREM(3):

The set of equations given by (1.11) to (1.14) are always satisfied by the Ricci tensor R_{jk} in a R- generalized recurrent Finsler spaces and special R- generalized recurrent Finsler spaces of order two of the first and second kinds.

CONCLUSION:

In this paper we have studied R-generalized recurrent Finsler spaces and special R-generalized recurrent Finsler space of the order two and we established some results in the form of theorems (1-3).

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