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S – and T– Matrix Formulations for 2–Port Devices

Siddharth Savyasachi Malu

Department of Astronomy, Astrophysics and Space Engineering, IIT Indore, India

Abstract:

We present the S– and T–matrix formulations of device characteristic for a 2–port device. We demonstrate that while the S–matrix formulation is intuitive and corresponds to physical quantities, the T–matrix formulation is better when it comes to handling multiple devices in series. We then compute the translation from S–matrix to T–matrix formulations and vice–versa.

Keywords: Electrical Engineering, S–matrix, T–matrix

1. Introduction: Two port devices and the S-matrix

Most of the microwave devices we use in our lab are 2-port devices, and are usually used in series, e.g. a w/g twist with a w/g straight piece. Any 2-port device has two possible inputs and two outputs. We label the inputs with a and outputs with b. For all practical purposes, we are interested not in the values of the outputs b, but what they are compared to the inputs. In other words, we wish to look at the generic ratios

$$\frac{output}{input} \tag{1}$$

for all four quantities.

The most simple-minded approach would be to define the four ratios $\frac{b_1}{a_1}$ etc., but we can write these out systematically as:

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{2}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{3}$$

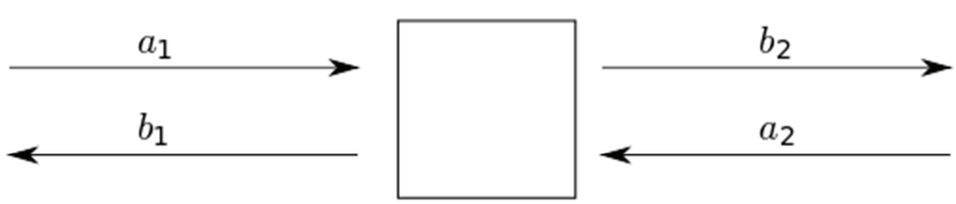

Figure 1: Schematic of the 2-port device

The above equations can also be in vector form written as:

$$\vec{b} = \vec{S}.\vec{a} \tag{4}$$

or, better still, in matrix form, which is more useful for our purpose:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (5)$$

Note for the mathematically inclined (a.k.a nerdy): each of the four quantities b_1, b_2, a_1, and a_2 is independent of all others, and so these are four linearly-independent quantities. This purely mathematical fact deduced from common-sense will help us later.

This definition of the so-called ¨S-matrix¨ is good-enough for anyone involved in making measurements, and the four S-parameters have the (by now obvious) meanings:

2. The need for a T-matrix

All this is fine for a single device, but what if there is a series of 2-port devices? Taking the familiar example from our lab of many waveguide devices in series, we see immediately that while we care about characterizing every single device, our eventual aim is not to slog away tediously trying to figure out how the input from one device becomes the output of another, but to figure out the effect of all the series devices at the same time.

Note, however, that this is not really possible with the S-matrix, since the inputs and also the outputs are on both sides of the device. Therefore, we need to change into a system where the inputs¨ are both on the left (right) and the outputs on the right (left). The simplest thing to do then would be to have this formalism worked out such that the net effect of all devices would be:

Neteffect = $device_1 \times device_2 \times device_3 \times \ldots \times device_n$

This is why we need the so-called ¨T-matrix¨. Here is how the formalism is defined: instead of going from ´input´ to ´output´ (this is what the Smatrix does), we want to go from ´left´ to ´right´. Recall that when we wanted to go from ´input´ to ´output´, we changed from the matrix $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Now look at fig 1. The two quantities on the left are a_1 and b_1, and the two quantities on the right are a_2 and b_2. So, very naively, we wish to go from $\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$ to $\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

And so we need a new matrix to go from the ¨left-vector¨ to the ¨right-vector¨. Schematically, we can write this as:

$$\overrightarrow{R\imath ght} = T^{\rightarrow} \cdot \overrightarrow{Left} \quad (6)$$

or, a little more clearly, as:

$$\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \quad (7)$$

3. Conversion between S- and T-matrix

When we make measurements of a device, it makes sense to think in terms of S-parameters, especially since those are what all network analyzers output. So, we need to figure out a way to change from S-parameters to T-parameters and back. Lets try to figure out the former first: Essentially, there are four equations we need to work with for the four T-parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (8)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (9)$$

$$a_2 = T_{11}a_1 + T_{12}b_1 \quad (10)$$

$$b_2 = T_{21}a_1 + T_{22}b_1 \quad (11)$$

Equations 9 and 11 imply

$$S_{21}a_1 + S_{22}a_2 = T_{21}a_1 + T_{22}(S_{11}a_1 + S_{12}a_2) \quad (12)$$

Similarly, equation 8 and 10 imply

$$a_2 = T_{11}a_1 + T_{12}(S_{11}a_1 + S_{12}a_2) \quad (13)$$

Equation 12 is

$$S_{21}a_1 + S_{22}a_2 = T_{21}a_1 + T_{22}S_{11}a_1 + T_{22}\, S_{12}a_2) \quad (14)$$

or, grouping terms with a_1 and a_2 separately:

$$a_2[S_{21} - T_{21} - T_{22}S_{11}] + a_2[S_{22} - T_{22}\, S_{12}] = 0 \quad (15)$$

Each of the two brackets must equal separately, since a_1 and a_2 are independent, so that the second bracket yields

$$S_{22} - T_{22}\, S_{12} = 0 \Longrightarrow T_{22} = \frac{S_{22}}{S_{12}} \quad (16)$$

Now substitute this value of T_{22}into the equation we get from the first bracket

$$S_{21} - T_{21} - T_{22}S_{11} = 0 \Longrightarrow T_{21} = S_{21} - T_{22}S_{11} \Longrightarrow T_{21} = \frac{S_{12}S_{21} - S_{22}S_{11}}{S_{12}} \quad (17)$$

Now look at equation 13, which reads

$$a_2 = T_{11}a_1 + T_{12}S_{11}a_1 + T_{12}S_{12}a_2 \quad (18)$$

As above, we collect terms with a_1 and a_2

$$a_2 = [1 - T_{12}S_{12}] - a_1\left[T_{11} + \frac{S_{11}}{S_{12}}\right] = 0 \quad (19)$$

Again, using the linear independence of a_1 and $a_2$we get from the first bracket

$$T_{12} = \frac{1}{S_{12}} \quad (20)$$

and from the second bracket

$$T_{11} + \frac{S_{11}}{S_{12}} = 0 \Longrightarrow T_{11} = -\frac{S_{11}}{S_{12}} \quad (21)$$

We can now write out our T-matrix in terms of elements of the S-matrix thus

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} -\frac{S_{11}}{S_{12}} & \frac{1}{S_{12}} \\ \frac{S_{12}S_{21} - S_{22}S_{11}}{S_{12}} & \frac{S_{22}}{S_{12}} \end{bmatrix} \quad (22)$$

It turns out that we can manipulate the same equations to express the S-matrix in terms of the T-matrix thus

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} -\frac{T_{11}}{S_{12}} & \frac{1}{T_{12}} \\ \frac{T_{12}T_{21} - T_{22}T_{11}}{T_{12}} & \frac{T_{22}}{T_{12}} \end{bmatrix} \quad (23)$$

Another note for the vector-space inclined: what we have done essentially is changed from the ¨Input-Output¨ basis to the ¨Right-Left¨ basis, and found the corresponding change in the transformation matrix.

4. Conclusions:

We have successfully shown a logical path to convert S-matricec into T-matrices. This is crucial in not just Electrical Engineering, but all related fields of Electronics, Radio Frequency Engineering, Radio Astronomy, High-frequency RADAR, and mm-wave instrumentation.

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